

Q1 Which of the following is an interpretation of  $((\neg P \Rightarrow Q) \wedge R)$  that makes the sentence true?

- a)  $P = 1, Q = 0, R = 0$
- b)  $P = 0, Q = 0, R = 1$
- c)  $P = 1, Q = 1, R = 0$
- d)  $P = 1, Q = 1, R = 1$

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# Solution

- Plugging in  $P = 1$ ,  $Q = 1$ ,  $R = 1$ , we have  $((\neg 1 \Rightarrow 1) \wedge 1)$  which is equivalent to  $(\neg 1 \Rightarrow 1)$  since  $(x \wedge 1 = x)$ . Then,  $(\neg 1 \Rightarrow 1)$  is equivalent to  $(0 \Rightarrow 1)$ . We then note that implication is always true when the premise (here  $\neg P$ ) is false. Thus, the overall sentence is TRUE or 1.

Q2. Let  $KB = \{A, B\}$  be a knowledge base.  
Which sentences does KB entail?

- a) A OR NOT B
- b) NOT B
- c) NOT A
- d) NOT A AND NOT B
- e) both a and c

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- Note  $A$  is in the KB and so is entailed. Also,  $A$  implies  $A \text{ OR } C$  for any other sentence  $C$ . Thus plugging in  $C = (\text{NOT } B)$ , we have  $A$  implies  $A \text{ OR NOT } B$ , and so the KB entails this sentence.

Q3. Which of the following sentences represents  $(P \Rightarrow (Q \Rightarrow R))$  in CNF?

- a)  $P \text{ OR } (Q \text{ AND } R)$
- b)  $P \text{ AND } Q \text{ OR NOT } R$
- c)  $\text{NOT } P \text{ OR } (\text{NOT } Q \text{ OR } R)$
- d)  $P \text{ OR } Q \text{ OR } (R \text{ AND NOT } Q)$

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- a)  $P \text{ OR } (Q \text{ AND } R)$
- b)  $P \text{ AND } Q \text{ OR NOT } R$
- c)  **$\text{NOT } P \text{ OR } (\text{NOT } Q \text{ OR } R)$**
- d)  $P \text{ OR } Q \text{ OR } (R \text{ AND NOT } Q)$



# Solution

- We need to get rid of the IMPLIES by converting them into AND, OR, and, NOT statements. Recall,  $P \Rightarrow B$  is logically equivalent to  $\text{NOT } P \text{ OR } B$ . Similarly,  $Q \Rightarrow R$  is equivalent to  $\text{NOT } Q \text{ OR } R$ . Thus, setting  $B = \text{NOT } Q \text{ OR } R$ , we have in total  $\text{NOT } P \text{ OR } (\text{NOT } Q \text{ OR } R)$  is an equivalent sentence made with only ORs of variables or negations of variables. Thus, it is in CNF (note we did not need ANDs in this case).