Informed Search

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Based on slides from Andrew Moore (https://www.autonlab.org/resources/tutorials), modified by Xiaojin Zhu (UW-Madison) and Anthony Gitter

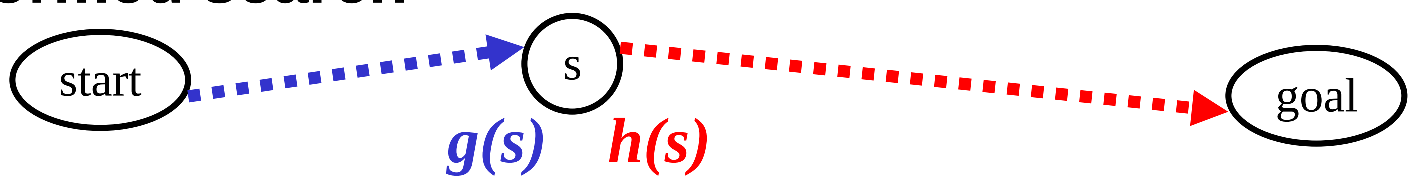
Main messages

- A* search. Always be optimistic.

Uninformed vs. informed search

- **Uninformed search** (BFS, UCS, DFS, IDS, etc.)
  - Knows the actual path cost ***g(s)*** from start to a node ***s*** in the fringe, but that's it.

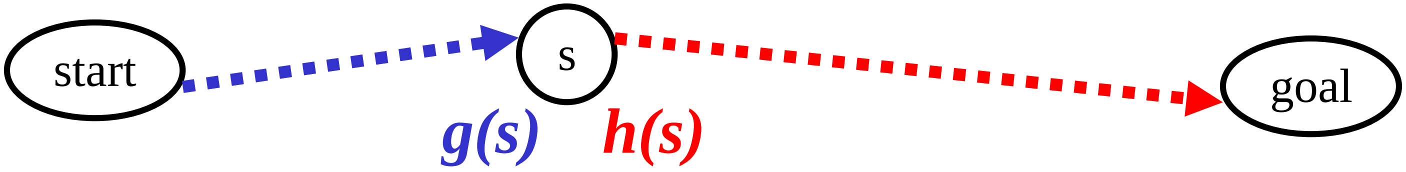
- **Informed search**



  - Also has a heuristic ***h(s)*** of the cost from ***s*** to goal.
    - ('h'= heuristic, non-negative)
  - Can be much faster than uninformed search.

Recall: Uniform-cost search

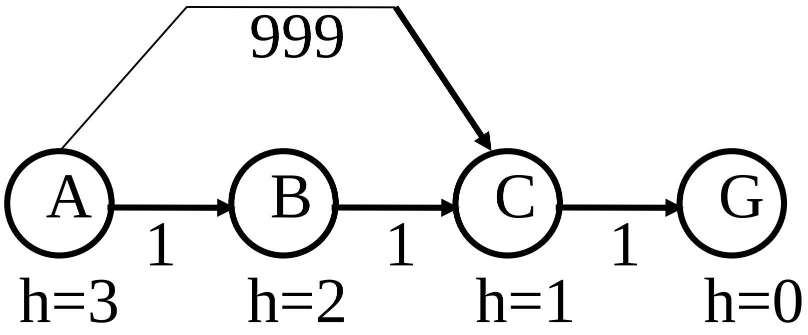
- Uniform-cost search: uninformed search when edge costs are not the same.
- Complete (will find a goal). Optimal (will find the least-cost goal).
- Always expand the node with the least ***g(s)***
  - Use a priority queue:
    - Push in states with their first-half-cost ***g(s)***
    - Pop out the state with the least ***g(s)*** first.
- Now we also have an estimate of the second-half-cost ***h(s)***, how to use it?



First attempt: Best-first greedy search

- Idea 1: use ***h(s)*** instead of ***g(s)***
- Always expand the node with the least ***h(s)***
  - Use a priority queue:
    - Push in states with their second-half-cost ***h(s)***
    - Pop out the state with the least ***h(s)*** first.
- Known as “best first greedy” search
- How’s this idea?

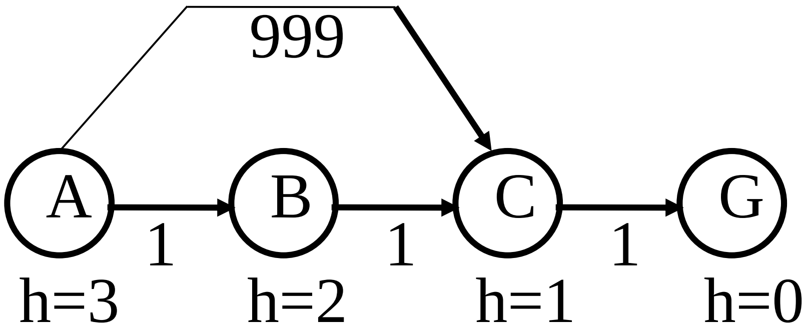
Best-first greedy search making bad decisions



- It will follow the path *A*→*C*→*G* (why?)
- Obviously not optimal
  - *A*→*B*→*C*→*G* is the optimal path

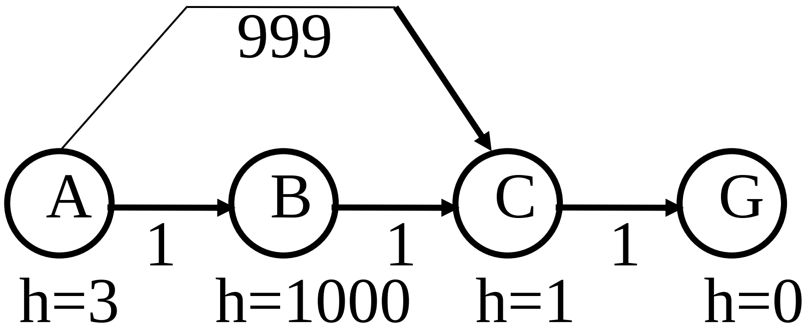
Second attempt: A search

- Idea 2: use ***g(s)***+***h(s)***
- Always expand the node with the least ***g(s)***+***h(s)***
  - Use a priority queue:
    - Push in states with their first-half-cost ***g(s)***+***h(s)***
    - Pop out the state with the least ***g(s)***+***h(s)*** first.
- Known as "A" search
- How's this idea?



- Works for this example

A search still not quite right



- A search is not optimal.

Third attempt: A* search

- Same as A search, but the heuristic function ***h()*** has to satisfy ***h(s)*** ≤ *h*(s)*, where *h*(s)* is the true cost from node *s* to the goal.
- Such heuristic function ***h()*** is called admissible.
  - An admissible heuristic never over-estimates

It is always optimistic

- A search with admissible ***h()*** is called A* search.
- Still require ***h(s)*** ≥ 0 as well

Admissible heuristic functions *h*

- 8-puzzle example

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- Which of the following are admissible heuristics?

  •h(n)=number of tiles in wrong position
  •h(n)=0
  •h(n)=1
  •h(n)=sum of Manhattan distance between each tile and its goal location

Admissible heuristic functions *h*

- 8-puzzle example

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- Which of the following are admissible heuristics?

  •h(n)=number of tiles in wrong position YES

  •h(n)=0 YES, uninformed uniform cost search

  •h(n)=1 NO, goal state

  •h(n)=sum of Manhattan distance between each tile and its goal location YES

Admissible heuristic functions *h*

- In general, which of the following are admissible heuristics? h*(n) is the true optimal cost from n to goal.
  - h(n)=h*(n)
  - h(n)=max(2,h*(n))
  - h(n)=min(2,h*(n))
  - h(n)=h*(n)-2
  - h(n)=sqrt(h*(n))

Admissible heuristic functions *h*

- In general, which of the following are admissible heuristics? h*(n) is the true optimal cost from n to goal.
  - h(n)=h*(n) YES
  - h(n)=max(2,h*(n)) NO
  - h(n)=min(2,h*(n)) YES
  - h(n)=h*(n)-2 NO, possibly negative
  - h(n)=sqrt(h*(n)) NO if h*(n)<1

Heuristics for Admissible heuristics

- How to construct heuristic functions?

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- Often by relaxing the constraints

•h(n)=number of tiles in wrong position

Allow tiles to fly to their destination in one step

•h(n)=sum of Manhattan distance between each tile and its goal location

Allow tiles to move on top of other tiles

“My heuristic is better than yours”

- A heuristic function h2 **dominates** h1 if for all s

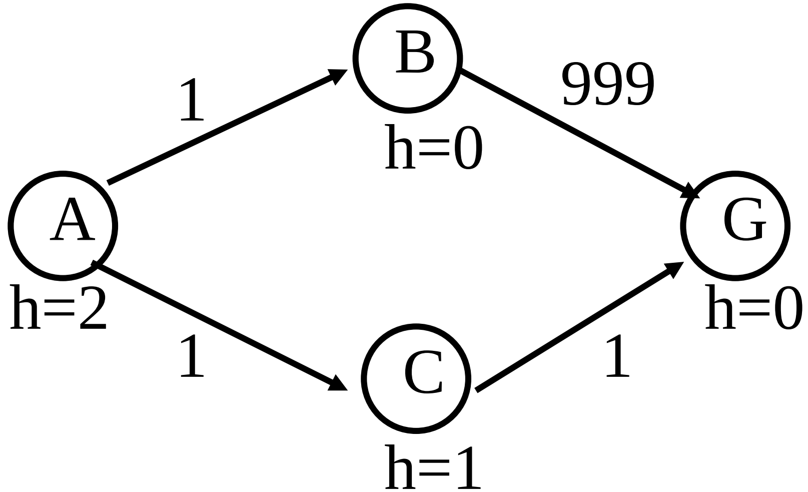
  $h1(s) \leq h2(s) \leq h^*(s)$
- We prefer heuristic functions as close to h* as possible, but not over h*.

But

- Good heuristic function might need complex computation
- Time may be better spent, if we use a faster, simpler heuristic function and expand more nodes

Q1: When should A* stop?

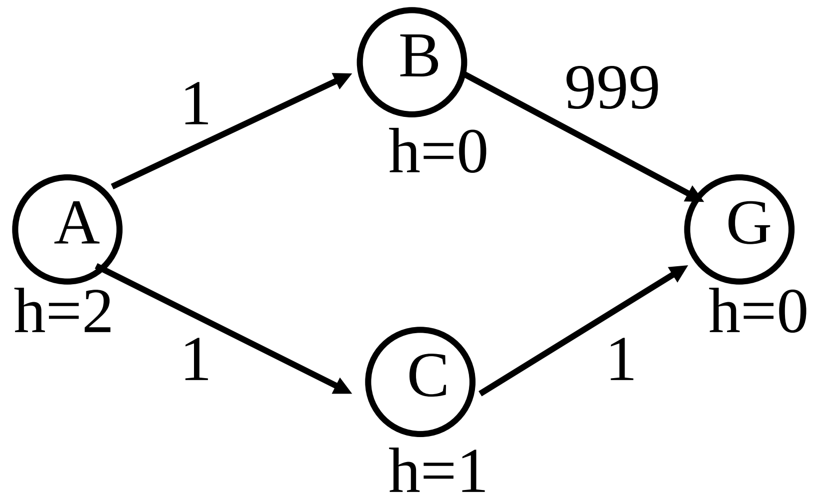
- Idea: as soon as it generates the goal state?



- h() is admissible
- The goal *G* will be generated as path *A*→*B*→*G*, with cost 1000.

Q1: The correct A* stop rule

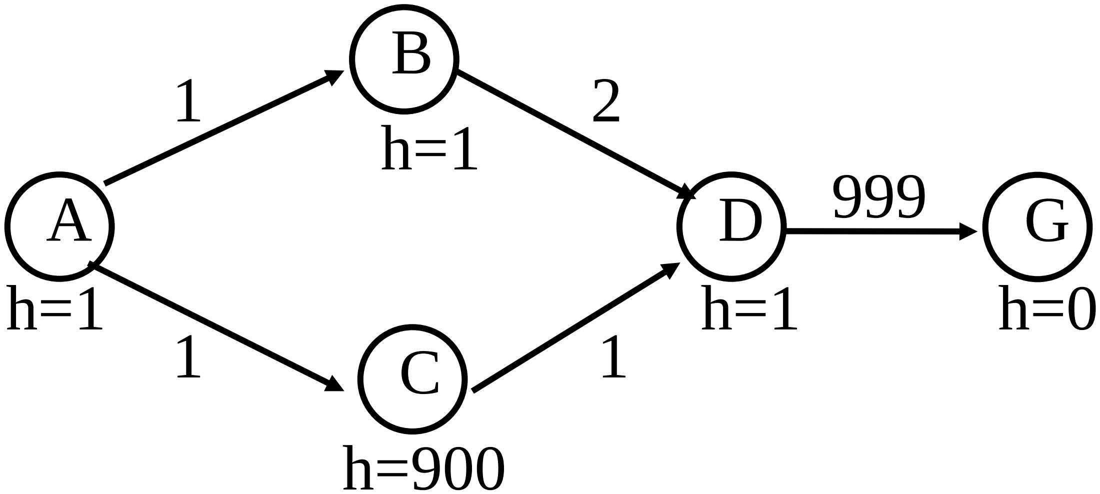
- A* should terminate only when a goal is popped from the priority queue



- If you have exceedingly good memory, you'll remember this is the same rule for uniform cost search on cyclic graphs.
- Indeed A* with h()≡0 is exactly uniform cost search!

Q2: A* revisiting expanded states (CLOSED)

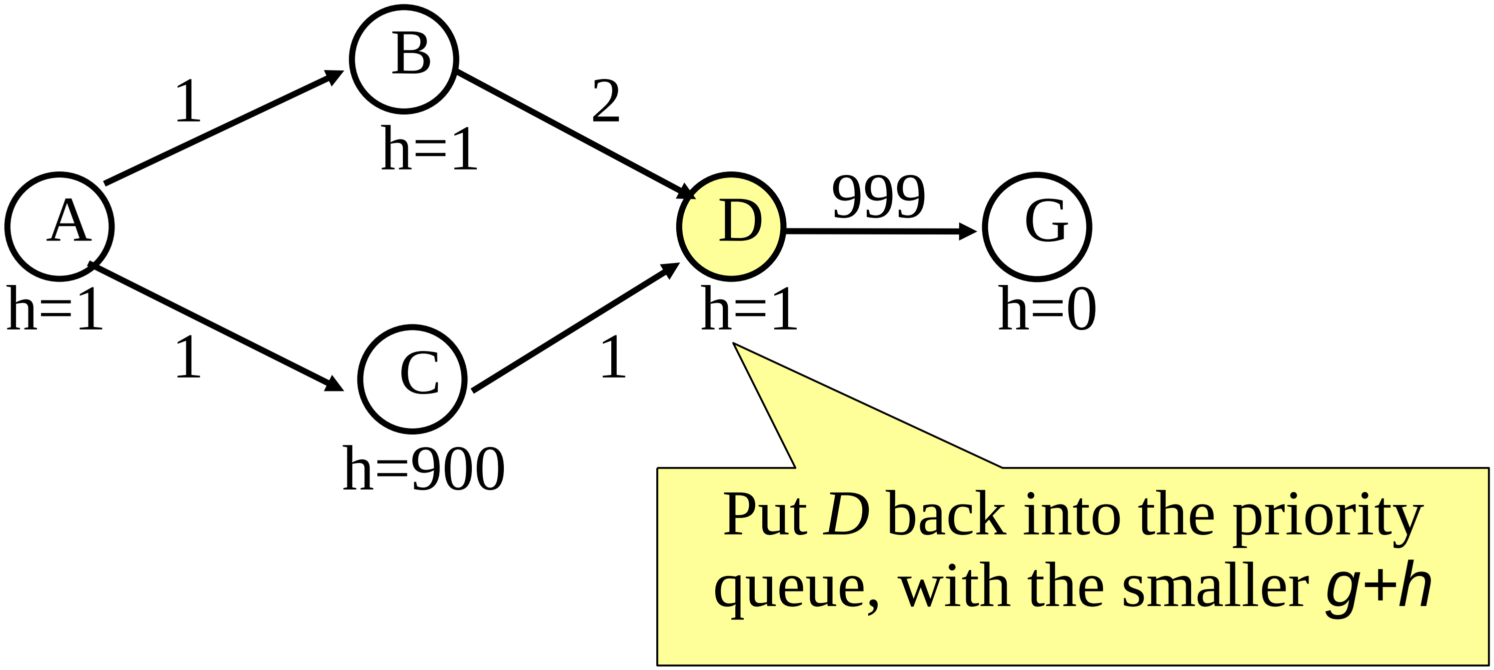
- One more complication: A* can revisit an expanded state and discover a shorter path



- Can you find the state in question?

Q2: A* revisiting expanded states (CLOSED)

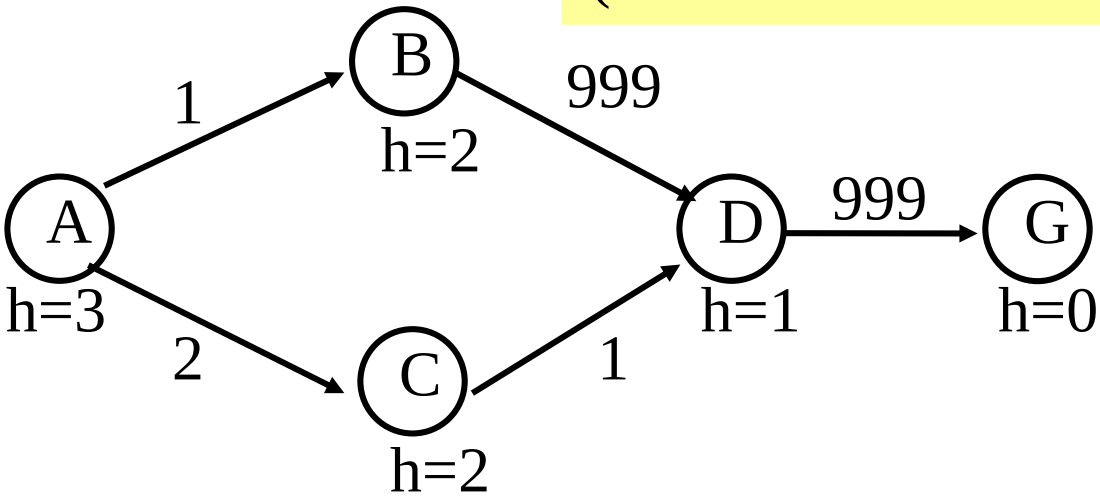
- One more complication: A* can revisit an expanded state and discover a shorter path



- Can you find the state in question?

Q3: What if A* revisits a state in the fringe (OPEN)?

(Note the numbers are different)



- ‘promote’ *D* in the queue with the smaller cost
- Uniform cost search would behave the same way

The A* algorithm

1. Put the start node S on the priority queue, called OPEN
2. If OPEN is empty, exit with failure
3. Remove from OPEN and place on CLOSED a node n for which f(n) is minimum (note that f(n)=g(n)+h(n))
4. If n is a goal node, exit (trace back pointers from n to S)
5. Expand n, generating all its successors and attach to them pointers back to n. For each successor n' of n
    1. If n' is not already on OPEN or CLOSED estimate h(n'), g(n')=g(n)+ c(n,n'), f(n')=g(n')+h(n'), and place it on OPEN.
    2. If n' is already on OPEN or CLOSED, then check if g(n') is lower for the new version of n'. If so, then:
        1. Redirect pointers backward from n' along path yielding lower g(n').
        2. Put n' on OPEN.
    3. If g(n') is not lower for the new version, do nothing.
6. Goto 2.

A*: the dark side

- A* can use lots of memory.
  O(number of states)
- For large problems
  A* will run out of memory
- We'll look at two alternatives:
  - IDA*
  - Beam search

Image: elite daily

IDA*: iterative deepening A*

- Memory bounded search. Assume integer costs
  - Do path checking DFS, do not expand any node with $f(n)>0$. Stop if we find a goal.
  - Do path checking DFS, do not expand any node with $f(n)>1$. Stop if we find a goal.
  - Do path checking DFS, do not expand any node with $f(n)>2$. Stop if we find a goal.
  - Do path checking DFS, do not expand any node with $f(n)>3$. Stop if we find a goal.

  … repeat this, increase threshold by 1 each time until we find a goal.
- This is complete, optimal, but more costly than A* in general.

IDA*: iterative deepening A*

- How many IDA* restarts?
  - Still assuming integer costs
  - Optimal solution cost C^*
  - At most C^* restarts

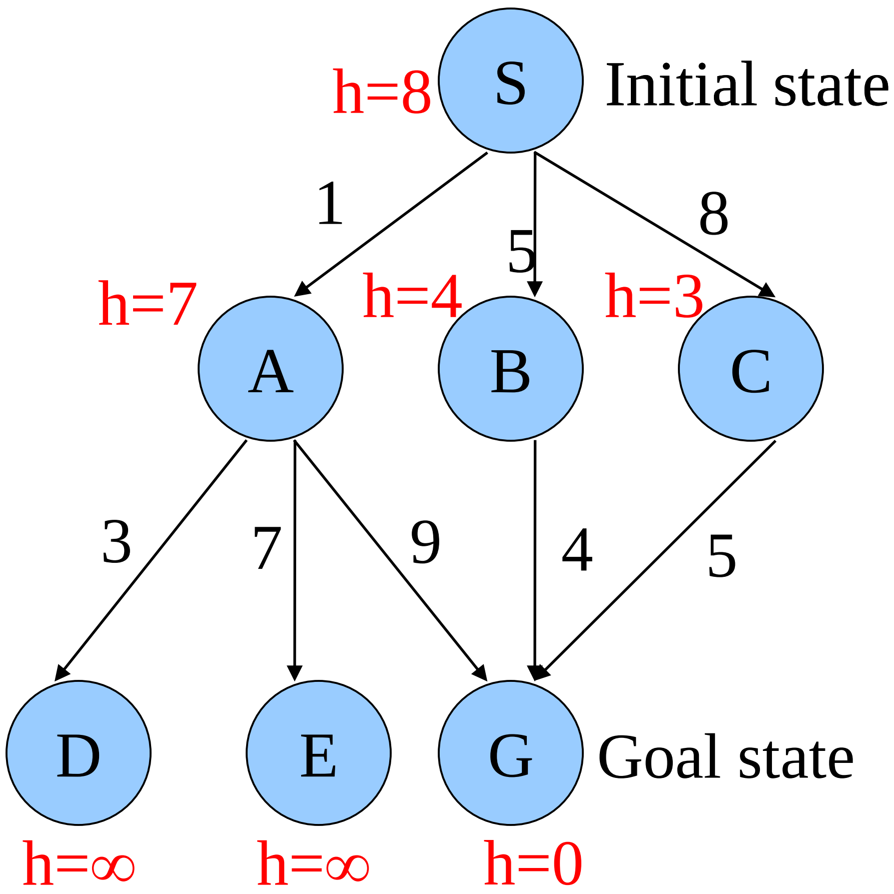
- What if we do not have integer costs?
  - Set initial threshold t
  - Do path checking DFS, do not expand any node with $f(n)>t$. Stop if we find a goal.
  - Set t to the min $f(n)$ of nodes that were not expanded
  - Restart
- Worst case requires restart for each state

Beam search

- Very general technique, not just for A*
- The priority queue has a fixed size *k*. Only the top *k* nodes are kept. Others are discarded.
- Neither complete nor optimal, nor can maintain an 'expanded' node list, but memory efficient.
- Variation: The priority queue only keeps nodes that are at most ε worse than the best node in the queue. ε is the beam width.
- Beam search used successfully in speech recognition.

Image: Nintendo World Report

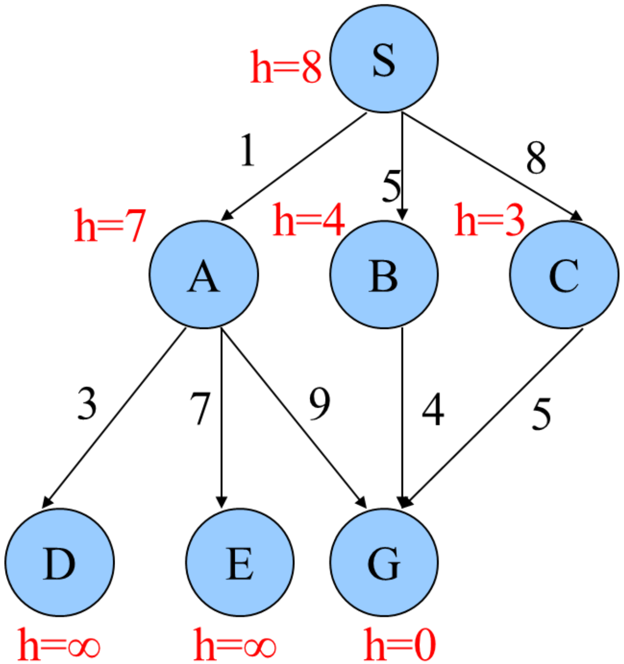
A* example



(All edges are directed, pointing downwards)

A* example

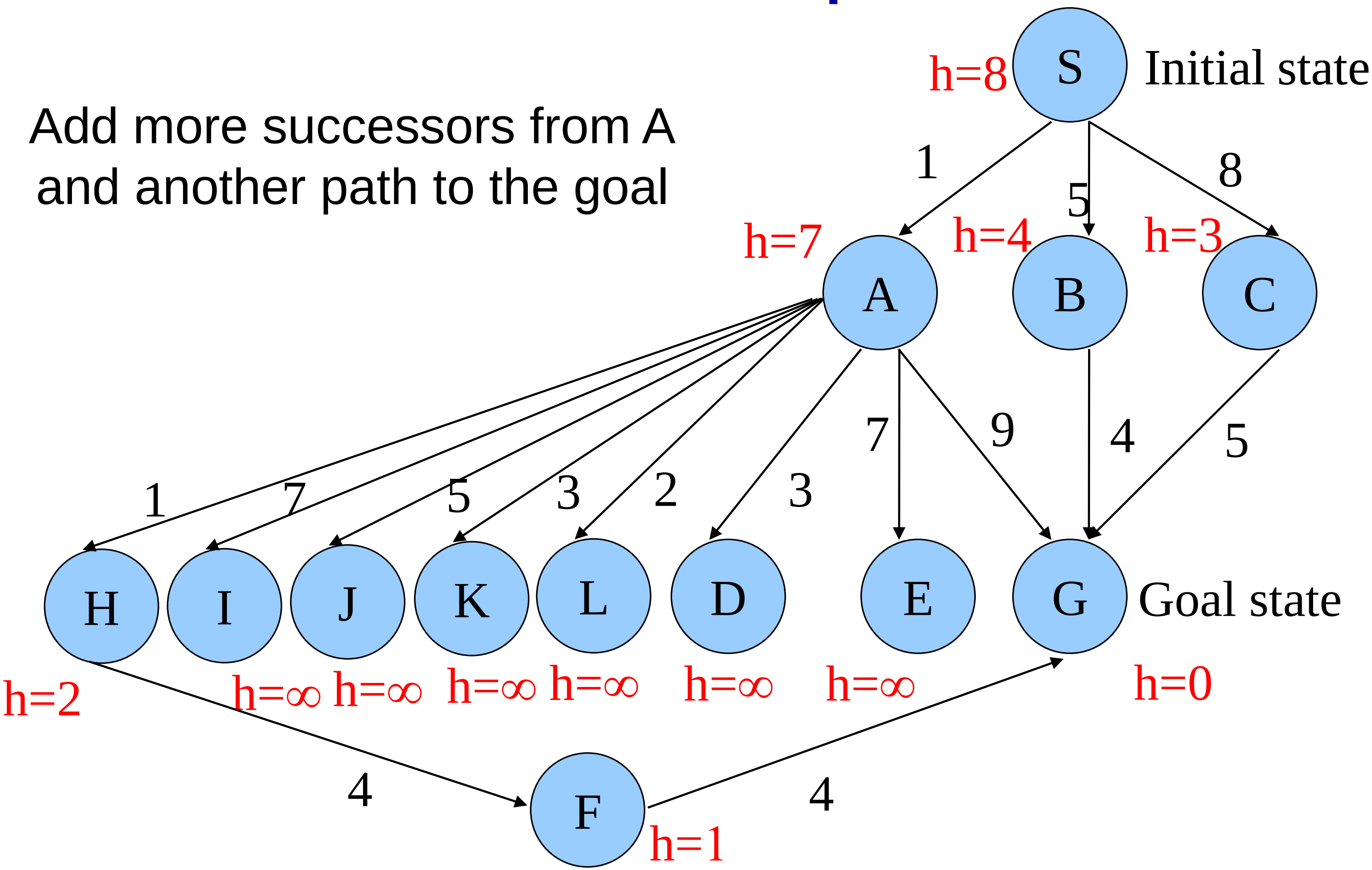
OPEN	CLOSED
S(0+8)	-
A(1+7) B(5+4) C(8+3)	S(0+8)
B(5+4) C(8+3) D(4+inf) E(8+inf) G(10+0)	S(0+8) A(1+7)
C(8+3) D(4+inf) E(8+inf) G(9+0)	S(0+8) A(1+7) B(5+4)
C(8+3) D(4+inf) E(8+inf)	S(0+8) A(1+7) B(5+4) G(9+0)



Backtrack: G => B => S

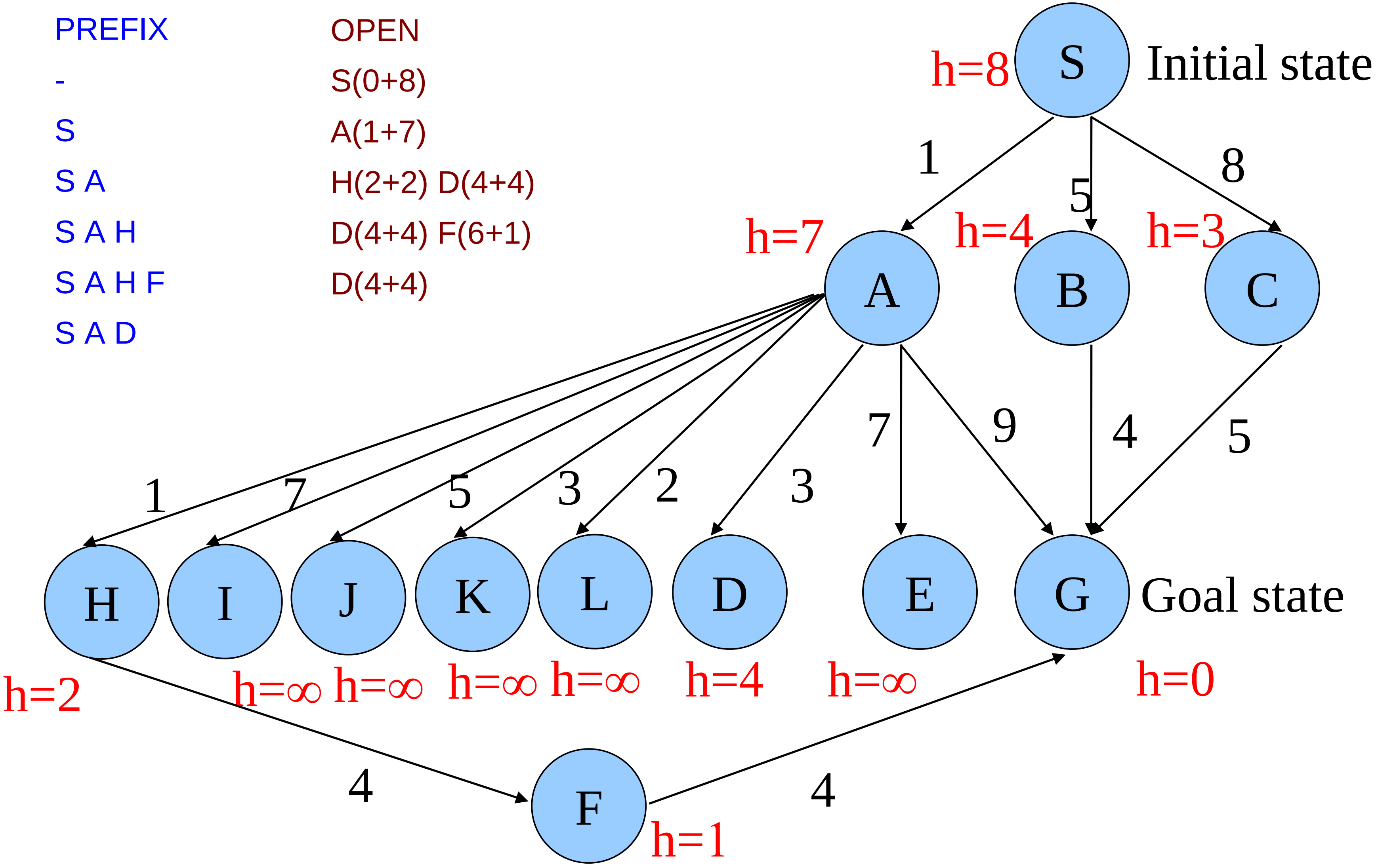
IDA* example

Add more successors from A
and another path to the goal



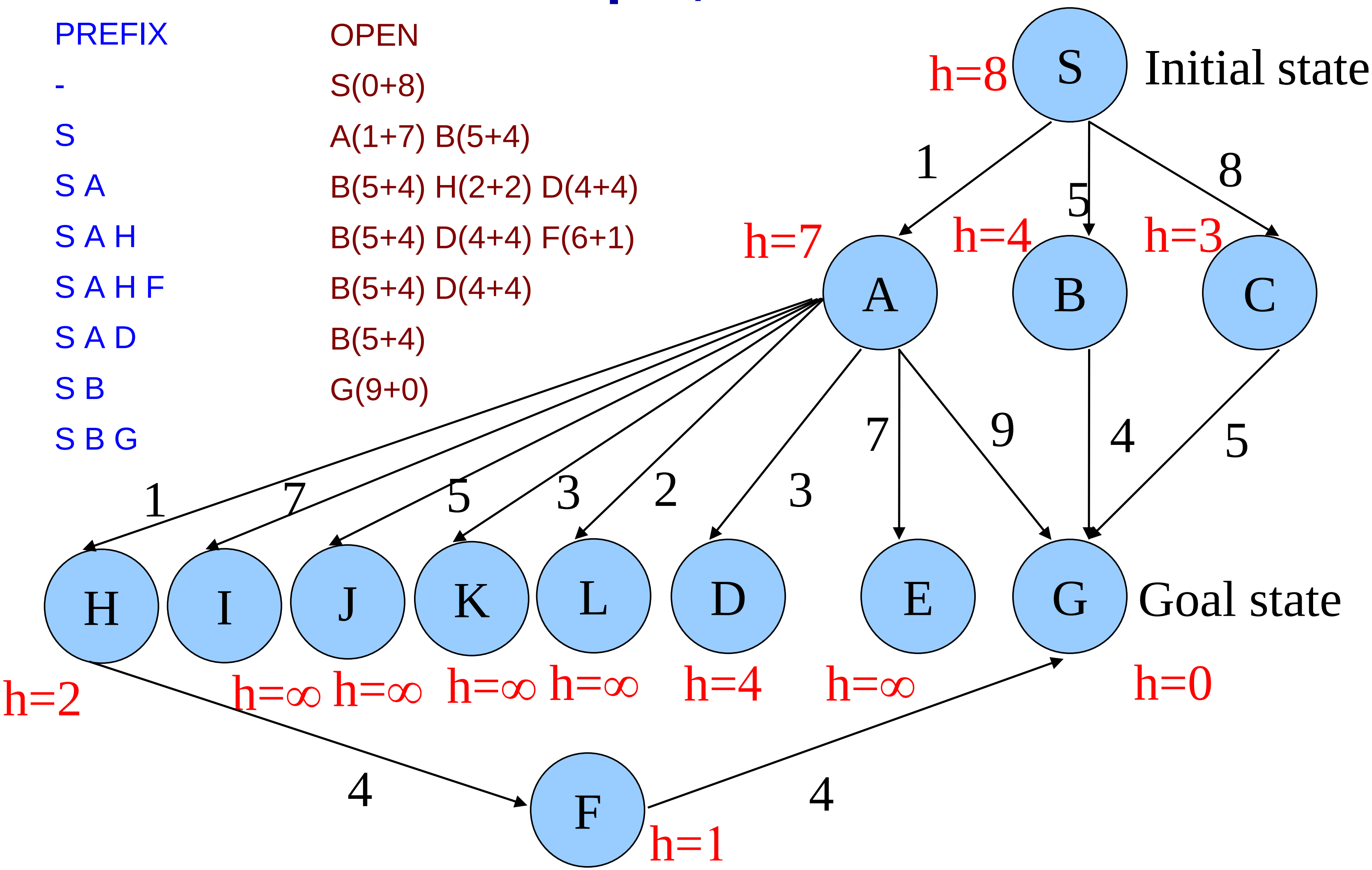
IDA* example, threshold *t* = 8

PREFIX	OPEN
-	S(0+8)
S	A(1+7)
S A	H(2+2) D(4+4)
S A H	D(4+4) F(6+1)
S A H F	D(4+4)
S A D	



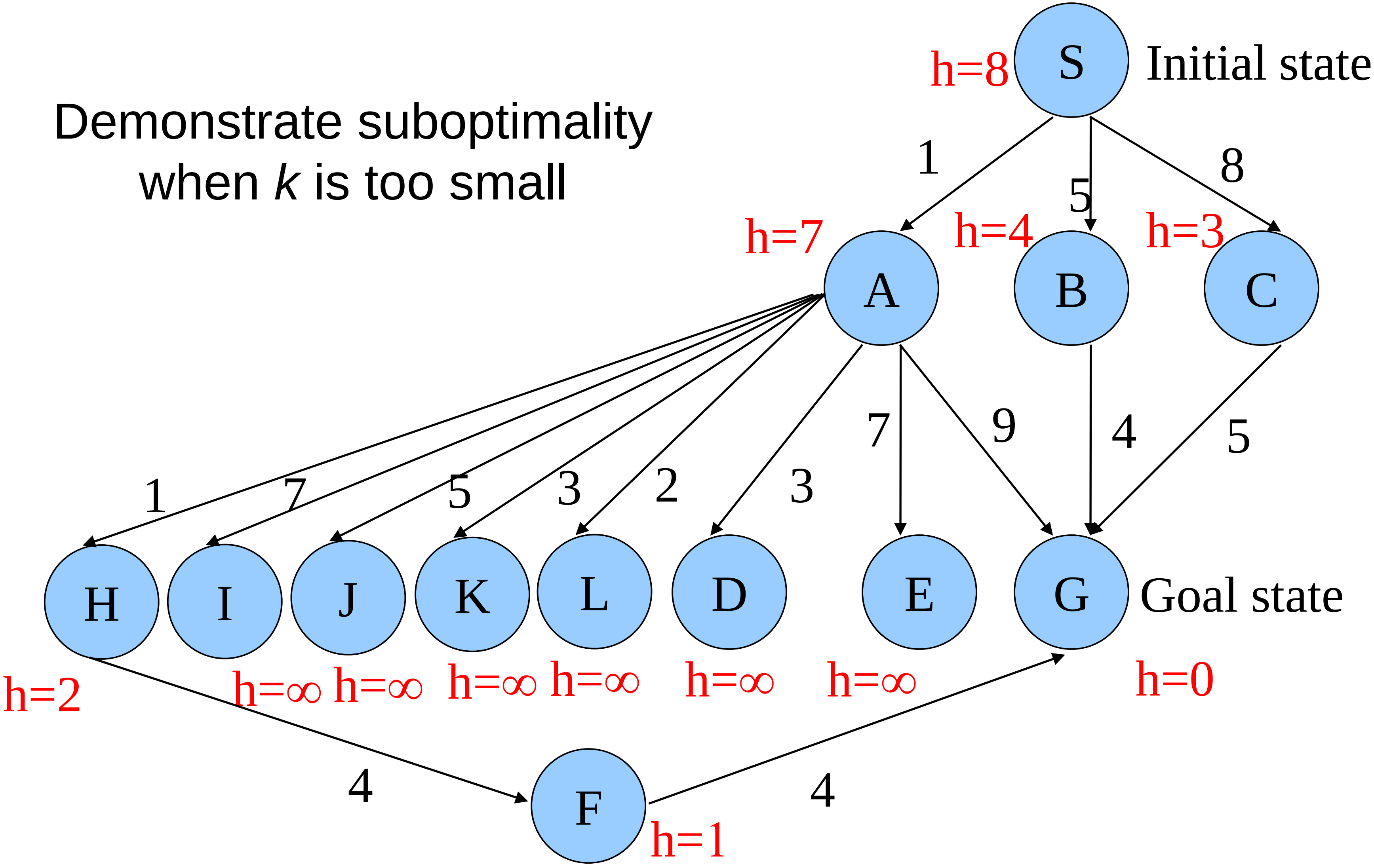
IDA* example, threshold *t* = 9

PREFIX	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
S A	B(5+4) H(2+2) D(4+4)
S A H	B(5+4) D(4+4) F(6+1)
S A H F	B(5+4) D(4+4)
S A D	B(5+4)
S B	G(9+0)
S B G	



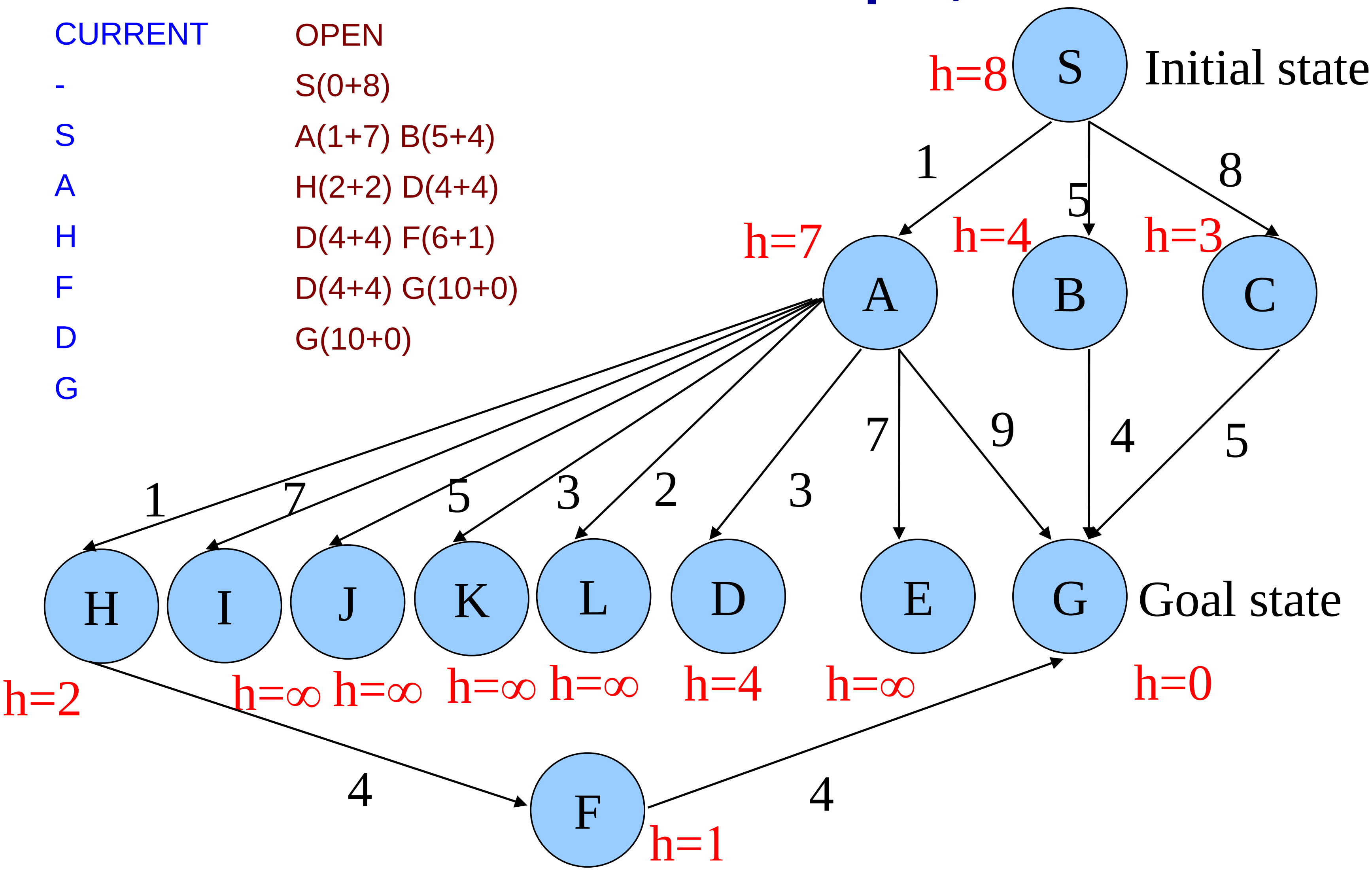
Beam search example

Demonstrate suboptimality when *k* is too small



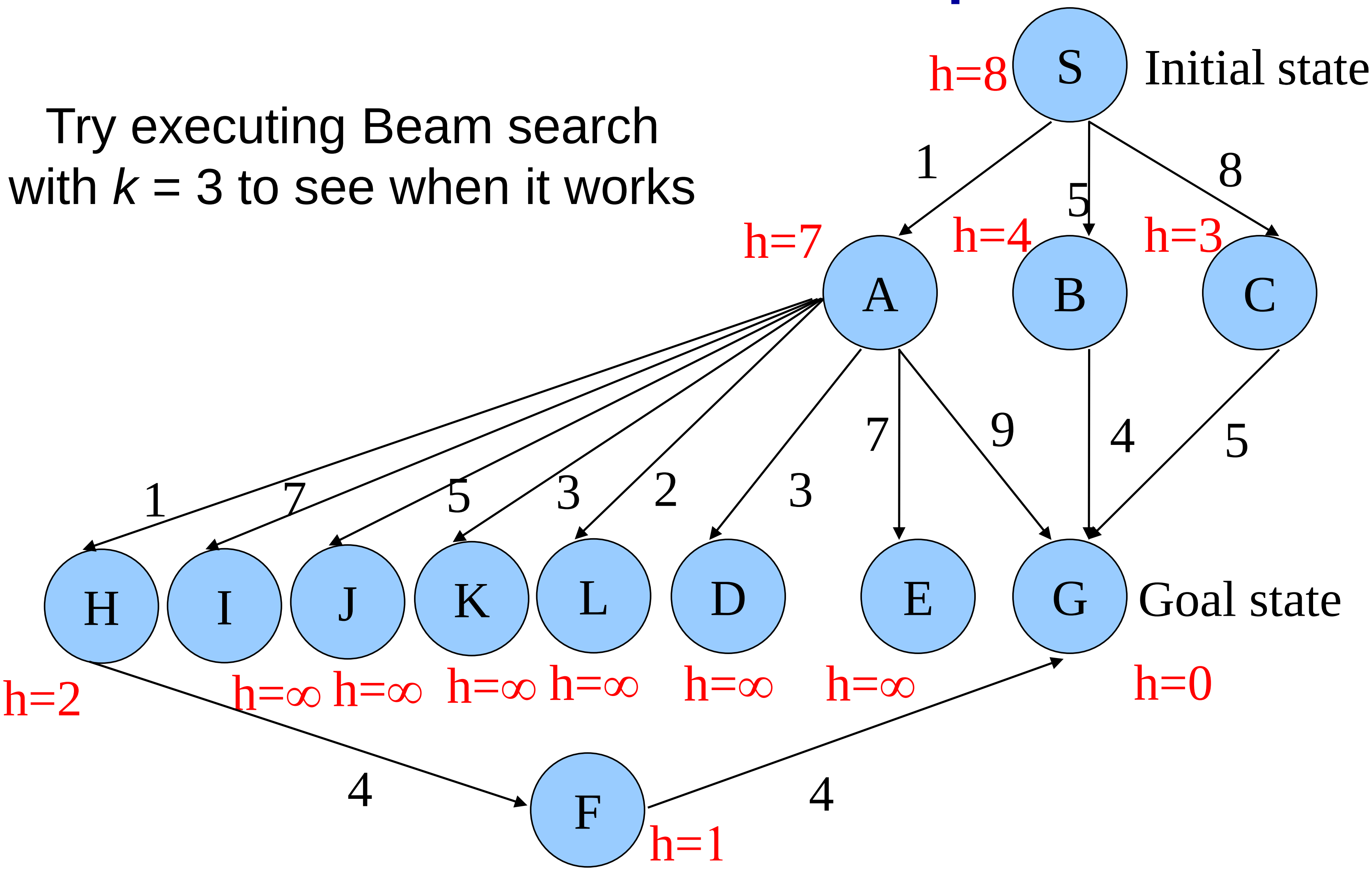
Beam search example, *k* = 2

CURRENT	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
A	H(2+2) D(4+4)
H	D(4+4) F(6+1)
F	D(4+4) G(10+0)
D	G(10+0)
G	



Beam search example

Try executing Beam search with *k* = 3 to see when it works



What you should know

- Know why best-first greedy search is bad
- Thoroughly understand A*
- Trace simple examples of A* execution
- Understand admissible heuristics
- Know how to improve A* space requirements

Appendix: Proof that A* is optimal

- Suppose A* finds a suboptimal path ending in goal G', where $f(G') > f^*$ = cost of optimal path
- Let's look at the first unexpanded node n on the optimal path (n exists, otherwise the optimal goal would have been found)
- $f(n) \geq f(G')$, otherwise we would have expanded n
- $f(n) = g(n)+h(n)$ by definition

  $= g^*(n)+h(n)$ because n is on the optimal path

  $\leq g^*(n)+h^*(n)$ because h is admissible

  $= f^*$ because n is on the optimal path
- $f^* \geq f(n) \geq f(G')$, contradicting the assumption at top